Design for Lifecycle Cost using Time-Dependent Reliability

Amandeep Singh
Zissimos P. Mourelatos
Jing Li

Mechanical Engineering Department
Oakland University
Rochester, MI 48309, USA

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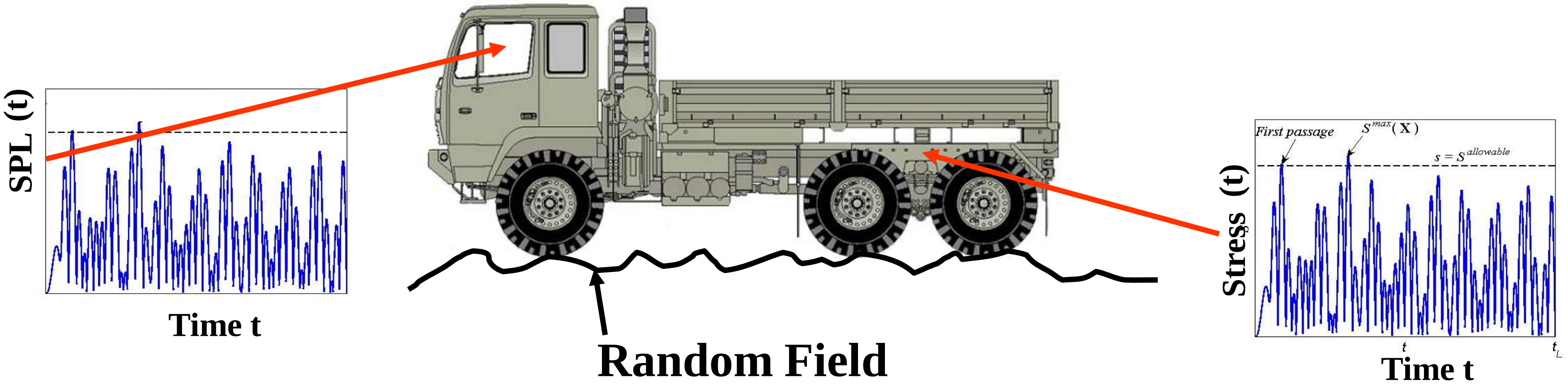
Problem Definition

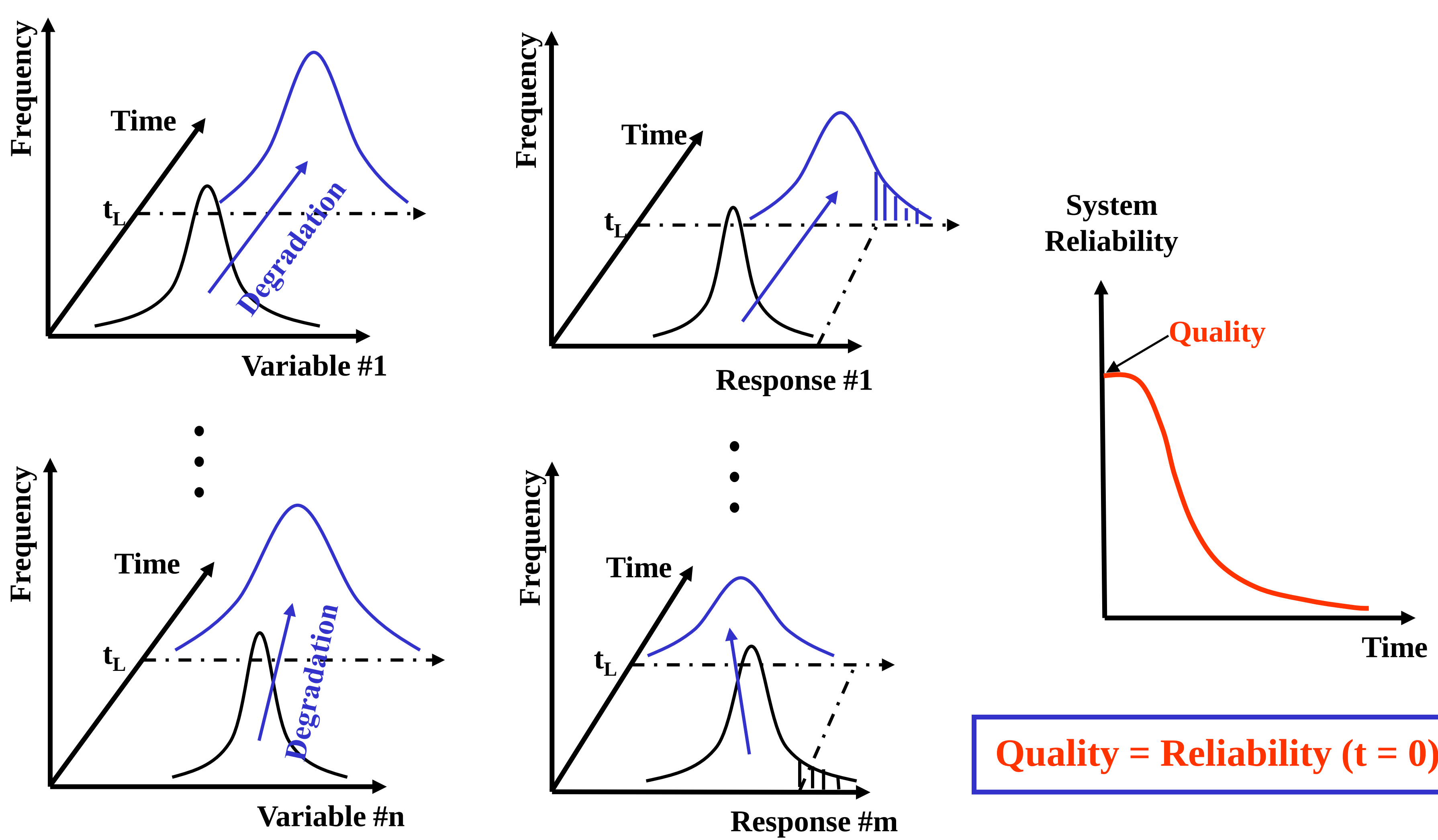


Response(t) = f [E(t), Degradation/Wear(t), Load(t)]

Random Process approach to reliability-based design is needed → time-dependent reliability

Problem Definition

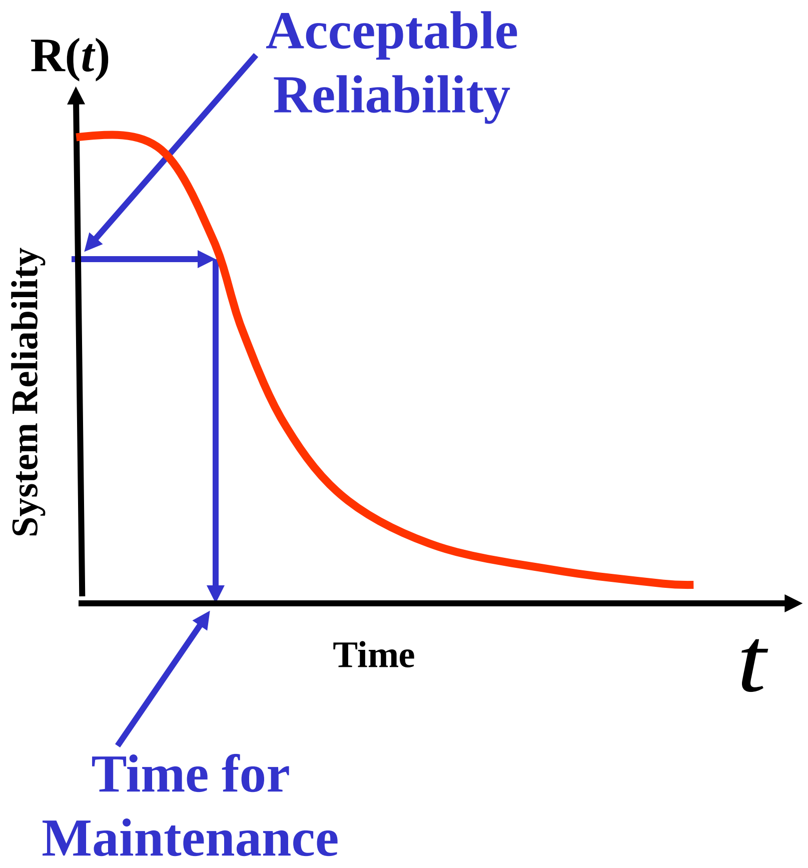
Input variables | System Responses | System Reliability



Quality = Reliability (t = 0)

What can we get from Time-Dependent Reliability?

- **Design for:**
  - **Lifecycle cost**
  - **Quality**
  - **Warranty**
  - **Maintenance schedule for CBM**



Design for Lifecycle Cost

Lifecycle Cost = Production Cost

+ Inspection Cost

+ Expected Variable Cost

Quality

Time-Dependent System Reliability

Accurate and efficient predictive tools are, therefore, needed to estimate **Time-dependent System Reliability**.

Design for Lifecycle Cost

$$C_L(\mathbf{d},\mathbf{X},t_f,r) = C_P(\mathbf{d},\mathbf{X}) + C_I(\mathbf{d},\mathbf{X},t_0) + C_V^E(\mathbf{d},\mathbf{X},t_f,r)$$

Lifecycle Cost — **Production Cost** — **Inspection Cost** — **Expected Variable Cost**

Final time — **Interest rate**

$$C_V^E(\mathbf{d},\mathbf{X},t_f,r) = \int_0^{t_f} c_F(t)\, e^{-rt} f_T^c(t)\, dt$$

Cost of failure at time t — **PDF of time to failure time**

$$F_T^c(t_L) = P\left(\exists t \in [0,t_L],\ such \quad that \quad g(\mathbf{X}(t),t) \le 0\right)$$

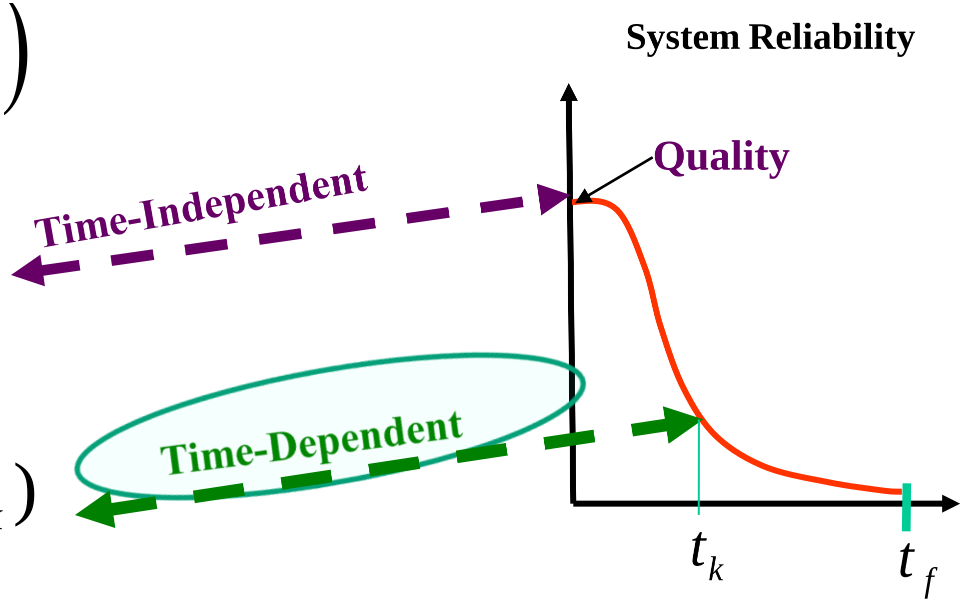
Problem Statement:
Design for Lifecycle Cost

$$\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}}} C_L\left(\mathbf{d},\boldsymbol{\mu}_{\mathbf{x}},t_f,r\right)$$

s. t.

$$F_{Q_i}^{i}\left(\mathbf{d},\mathbf{X},t_0\right)\le p_f\left(t_0\right)$$

$$F_{R}^{c}\left(\mathbf{d},\mathbf{X},t_k\right)\le p_f\left(t_k\right)$$



$$\mathbf{d}_L\le\mathbf{d}\le\mathbf{d}_U$$

$$\boldsymbol{\mu}_{\mathbf{X}_L}\le\boldsymbol{\mu}_{\mathbf{X}}\le\boldsymbol{\mu}_{\mathbf{X}_U}$$

Quantification of **time-dependent reliability** is a major challenge in this research.

Definitions / Observations

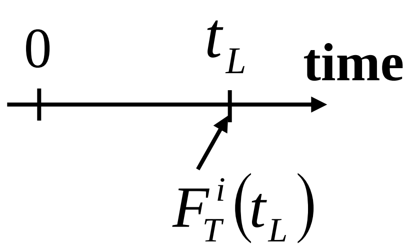
Reliability: **Ability of a system to carry out a function in a time period [0, t_L]**

$$p_f^c = P\left(t \le t_L\right) = F_T^c\left(t_L\right)$$ **Prob. of Time to Failure**

$$\boxed{F_T^c\left(t_L\right) = P\left(\exists t \in \left[0, t_L\right], such \quad that \quad g\left(\mathbf{X}\left(t\right), t\right) \le 0\right)}$$ **Cumulative Prob. of Failure**

$$\boxed{F_T^i\left(t_L\right) = P\left(g\left(\mathbf{X}\left(t_L\right), t_L\right) \le 0\right)}$$ **Instantaneous Prob. of Failure**

Time-Invariant Reliability | **Time-Variant Reliability**



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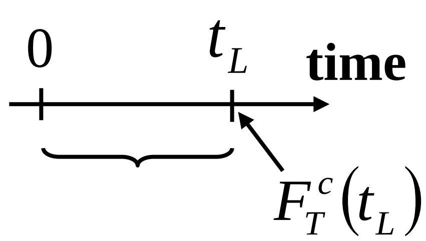
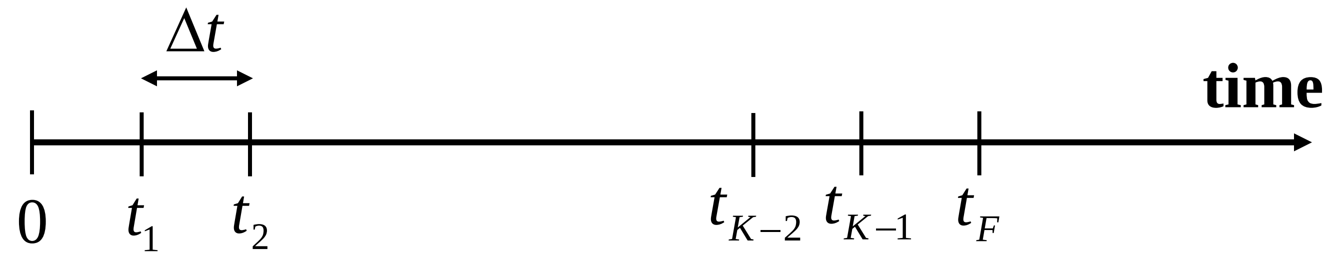


Calculation of Cumulative Probability of Failure

- **State-of-the-art Approaches**

  - **PHI2 method (Andrieu-Renaud, et al., *RESS*, 2004)**
  - **Set-Based approach (Son and Savage, *Quality & Rel. Engin.*, 2007)**



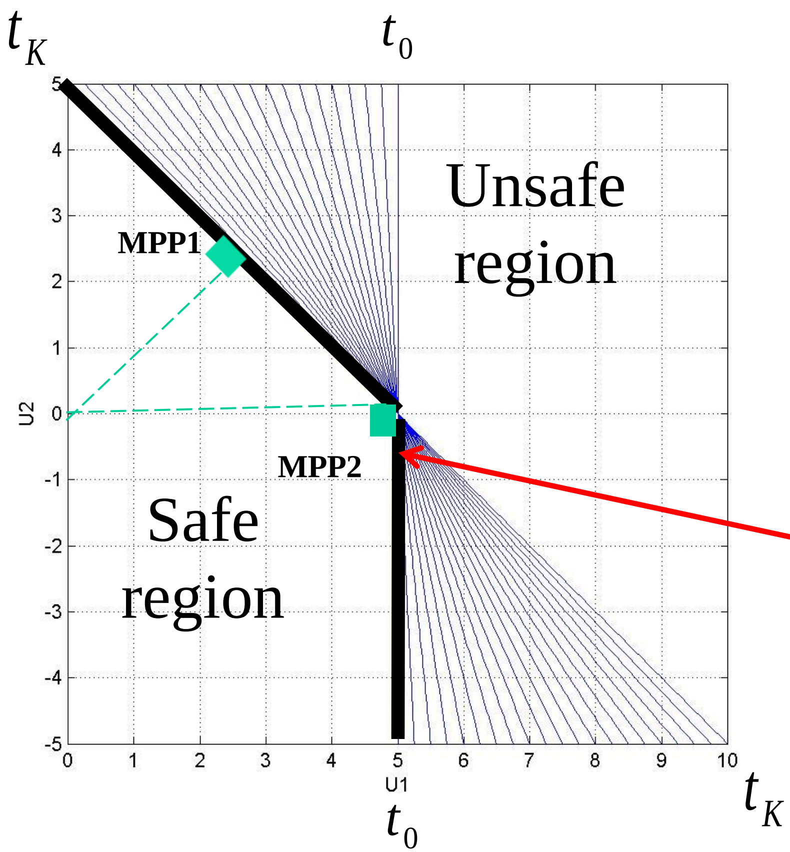
$$t_F = K\Delta t$$

- **State-of-the-art approaches are in general, inaccurate due to:**

  - **Choice of** Δt
  - **Not including contribution of <u>all</u> discrete times**

Cumulative Probability of Failure

Composite Limit State



Example 1: Linear Limit State

$$\boxed{g(X_1, X_2, t) = X_1 + tX_2}$$

$$X_1 \sim N(-5, 1^2) \quad X_2 \sim N(0, 1^2)$$

$$t_0 < t \le t_K$$

"Composite" limit state

$$\boxed{F_T^c(t_F) = P\left(\bigcup_{k=0}^{K} g(\mathbf{X}(t_k), t_k)\right)}$$

Single MPP of instantaneous limit states evolves into multiple MPPs of composite limit state.

Cumulative Probability of Failure

Composite Limit State

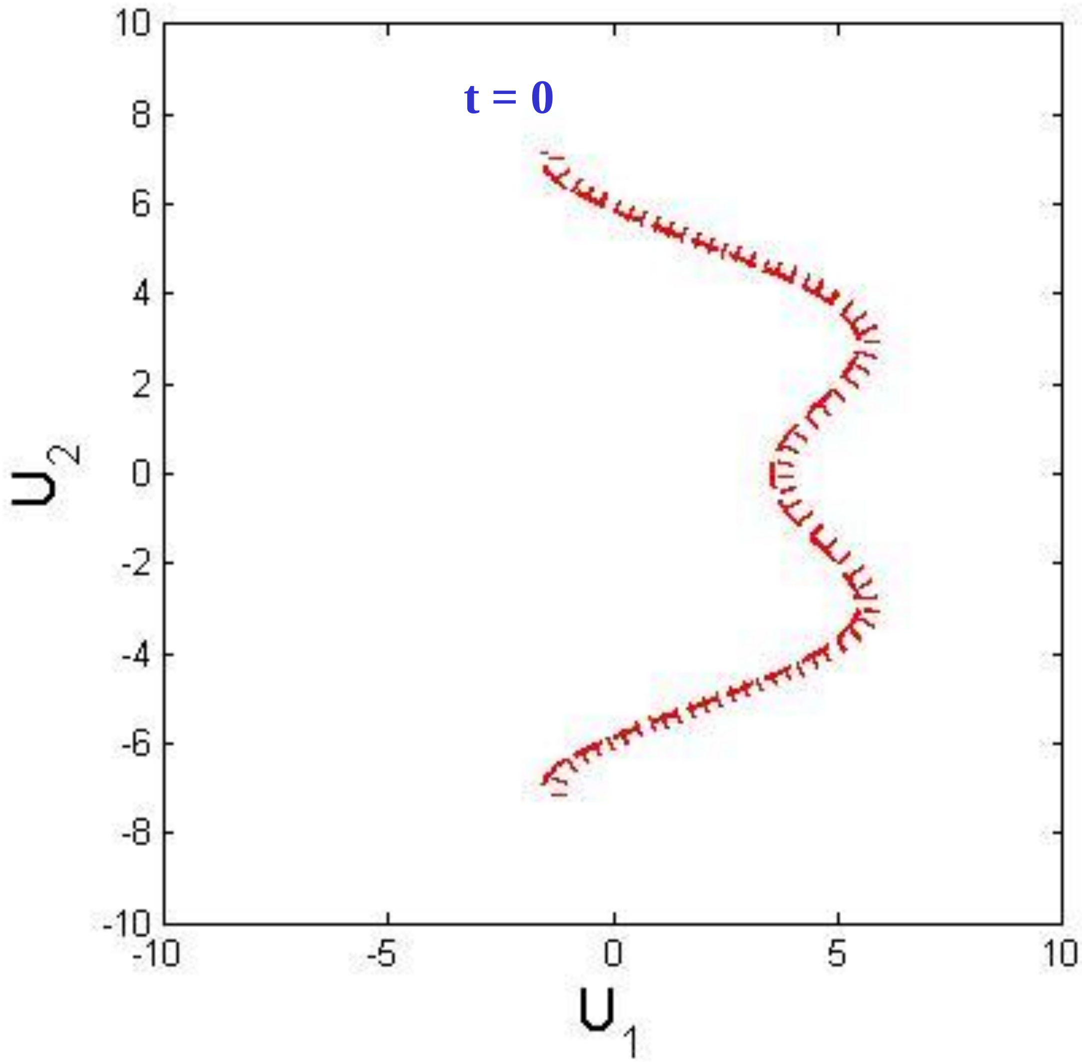
Example 2:

$$\boxed{g(X_1, X_2) = 1 - \frac{X_1 - 1000 X_2 \sin(4\pi t + X_2)\alpha}{12000\alpha}, \quad \alpha = 52966}$$

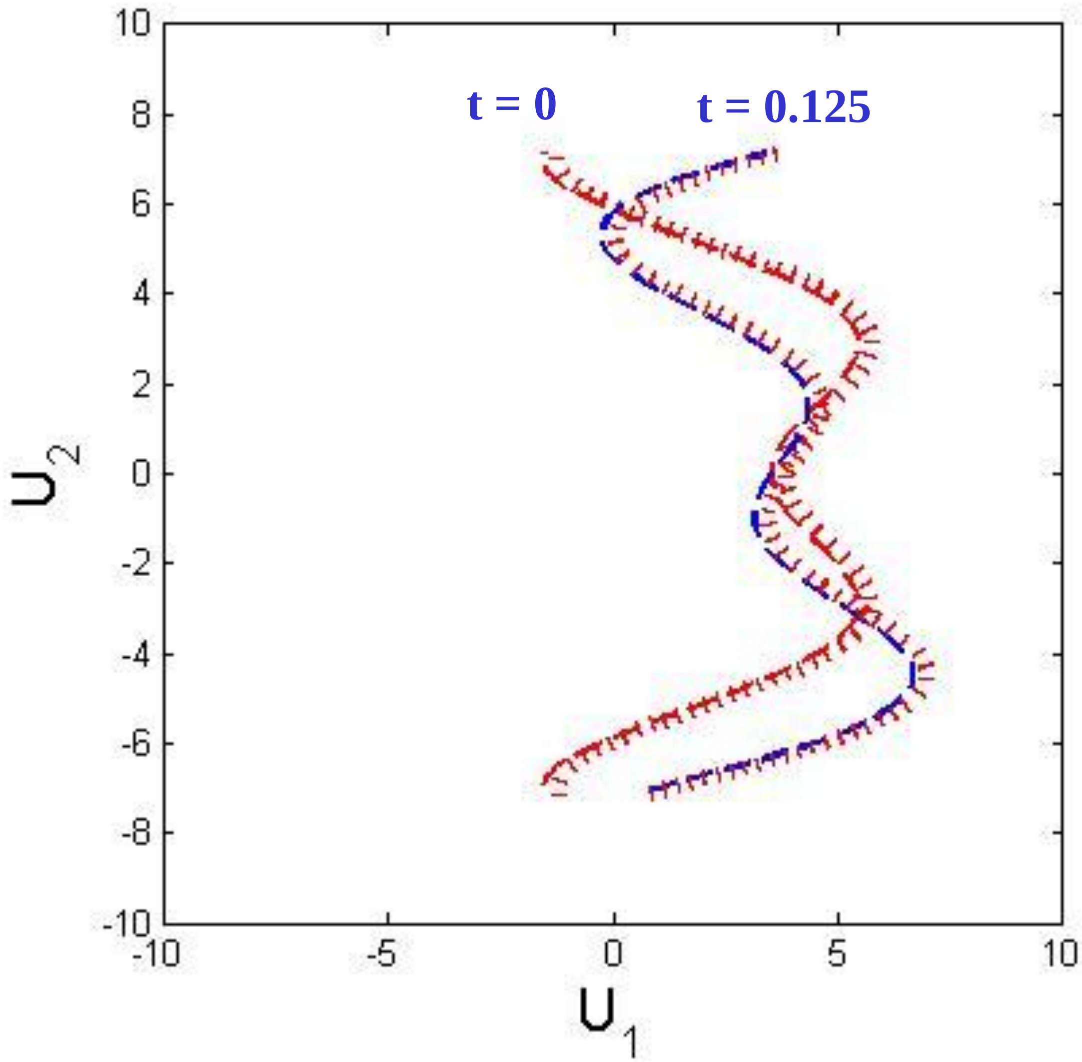
$$X_1 \sim N\left(4.58E08,\ (5E07)^2\right)$$

$$X_2 \sim N\left(0,\ 0.7^2\right)$$

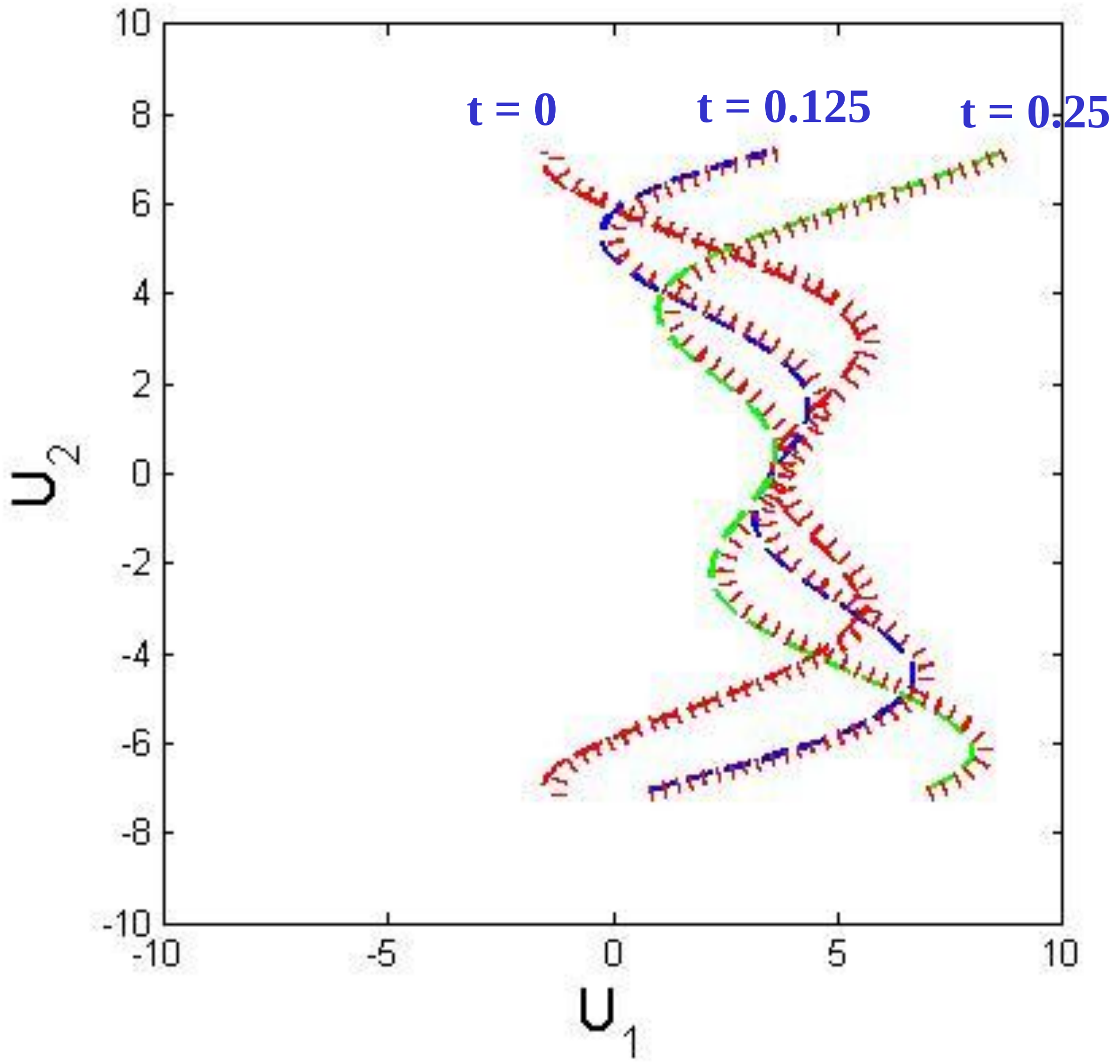
Composite Limit State: Example 2



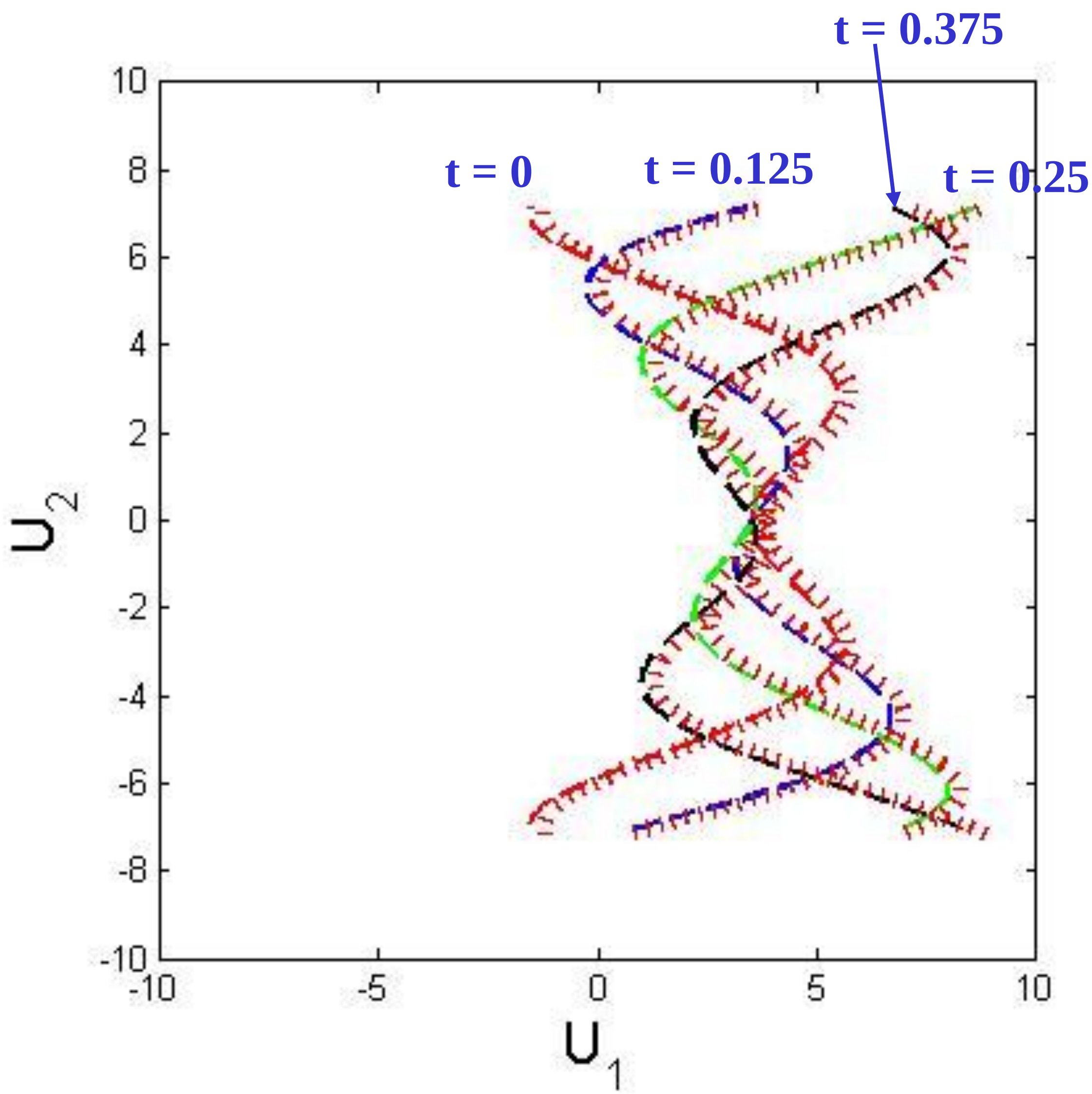
Composite Limit State: Example 2



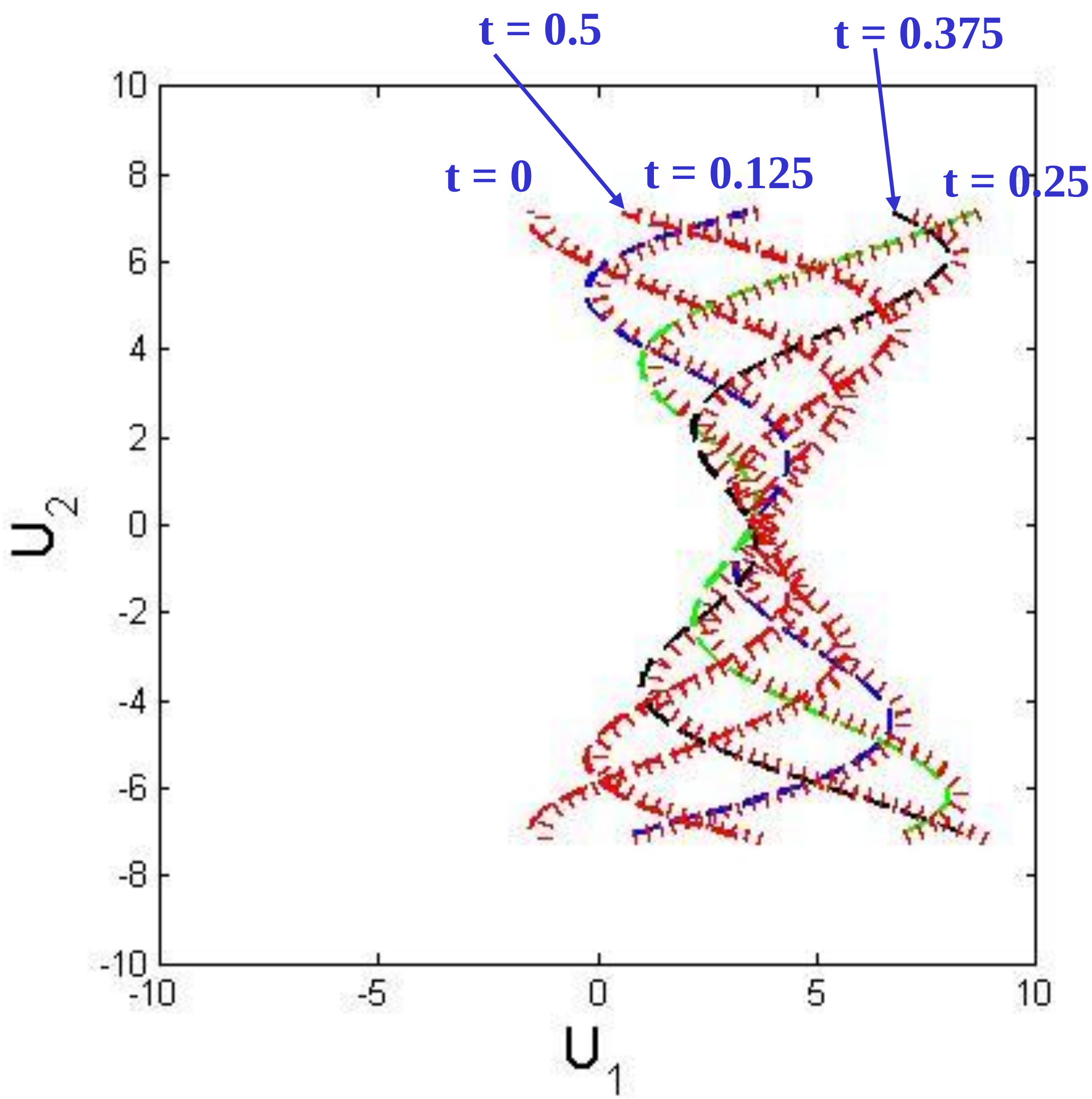
Composite Limit State: Example 2



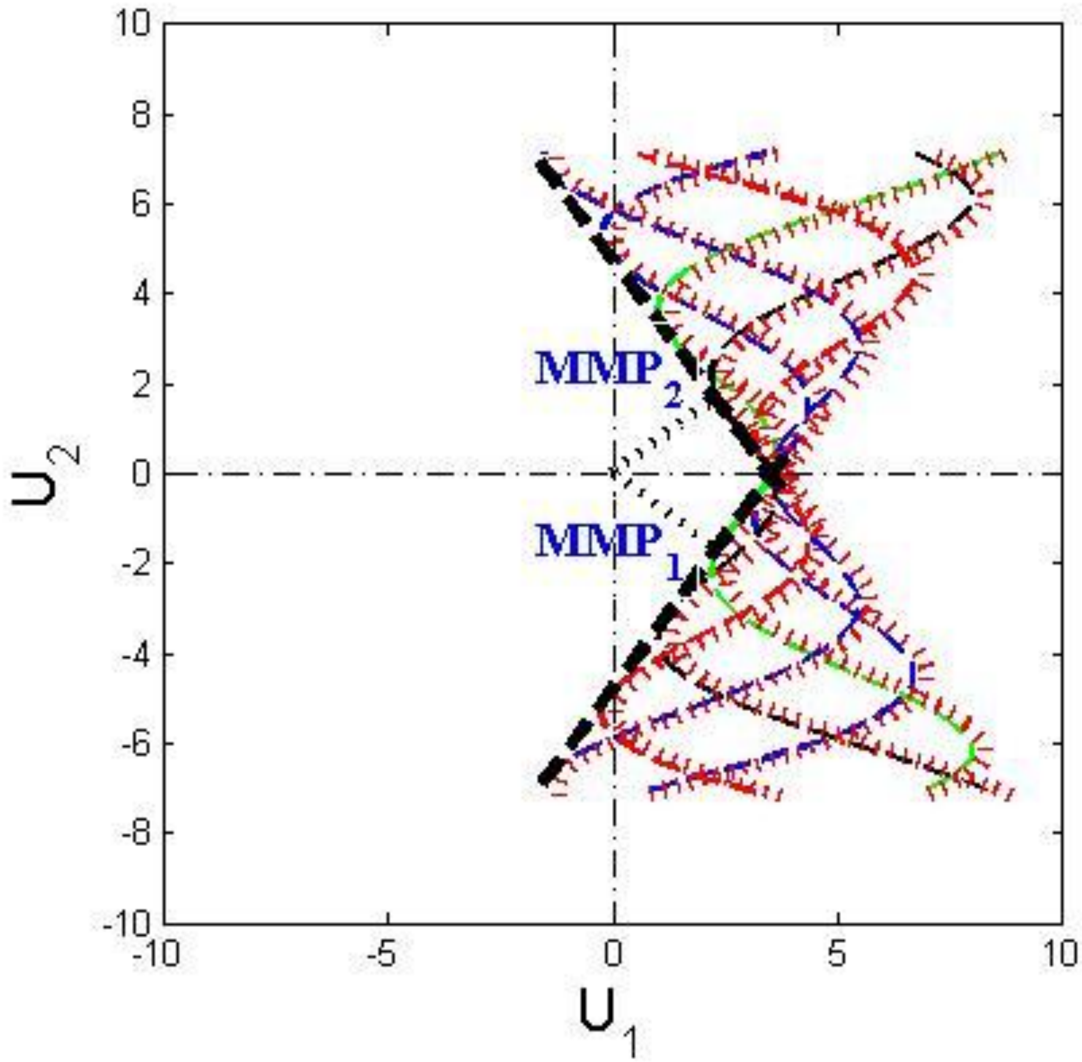
Composite Limit State: Example 2



Composite Limit State: Example 2



Composite Limit State: Example 2



Calculation of Probability of Failure
Two Proposed Approaches

- Reliability Index Approach
  - – Limit State is kept Time-dependent i.e. $g(\mathbf{d}, \mathbf{X}, t) = 0$

- Maximum Response Approach
  - – Limit State is converted into Time-Independent i.e $g(\mathbf{d}, \mathbf{X}) = 0$

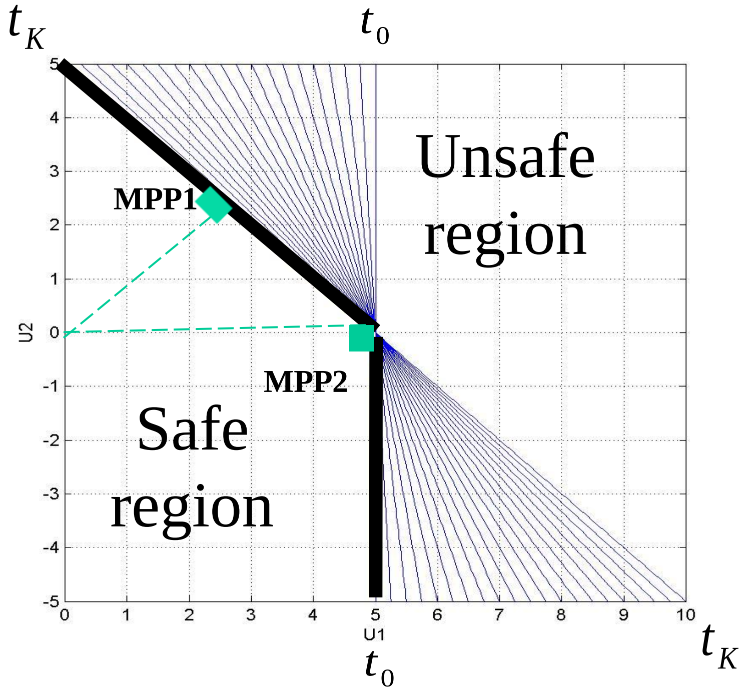
Calculation of Probability of Failure

- **Reliability Index Approach:**

$$\beta = \min_{\mathbf{U},t} \left\| \mathbf{U} \right\|_2$$

$$s.t. \quad g(\mathbf{U},t) = 0$$

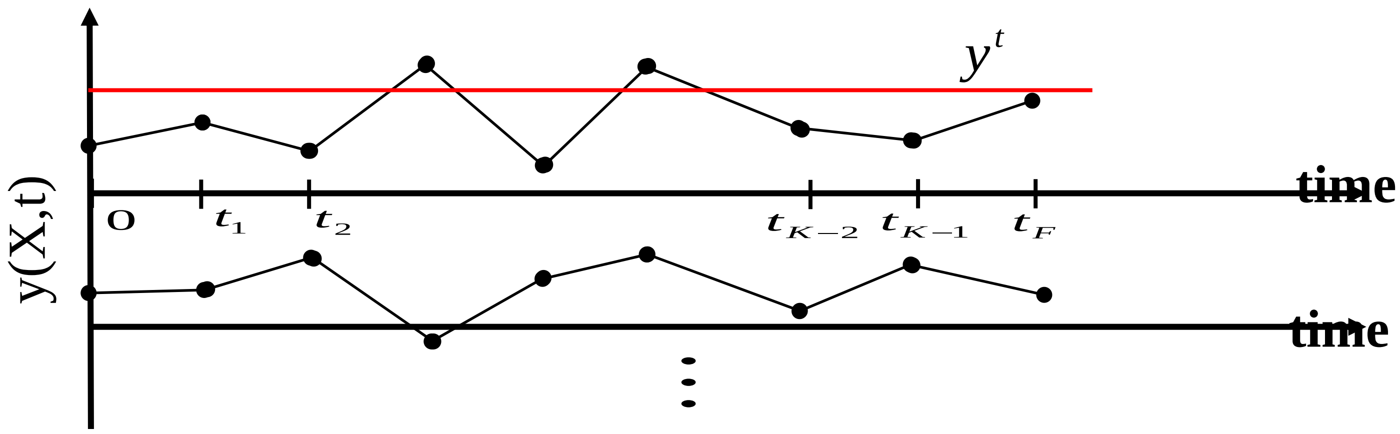
$$t_0 \le t \le t_{max}$$



Time is treated as an additional design variable in RIA optimization.

Cumulative Probability of Failure

➢ **Maximum Response Approach:**



$$y^{\max}(\mathbf{d},\mathbf{X}) = \max_{t_{\min} \le t \le t_{\max}} y(\mathbf{d},\mathbf{X},t)$$

$$\boxed{F_T^c(t_F) \;\; = P(y^{\max}(\mathrm{X}) > y^t) \;\; = P(y^t - y^{\max}(\mathrm{X}) < 0)}$$

Composite Limit-State as time-independent is defined as:

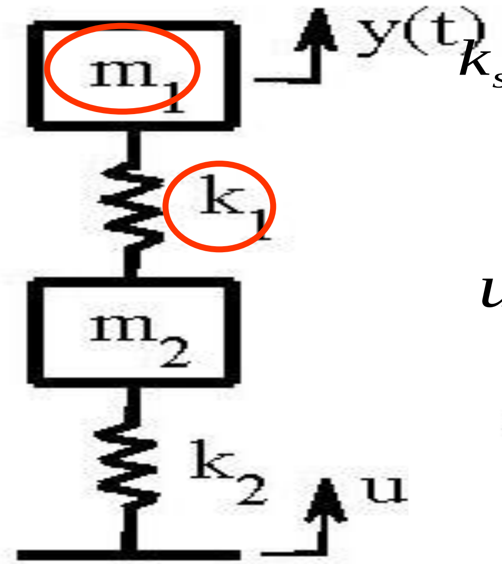
$$g(\mathbf{d},\mathbf{X}) = y^t - y^{\max} \le 0$$

Calculation of p_f :

Two-DOF System

Two-DOF System

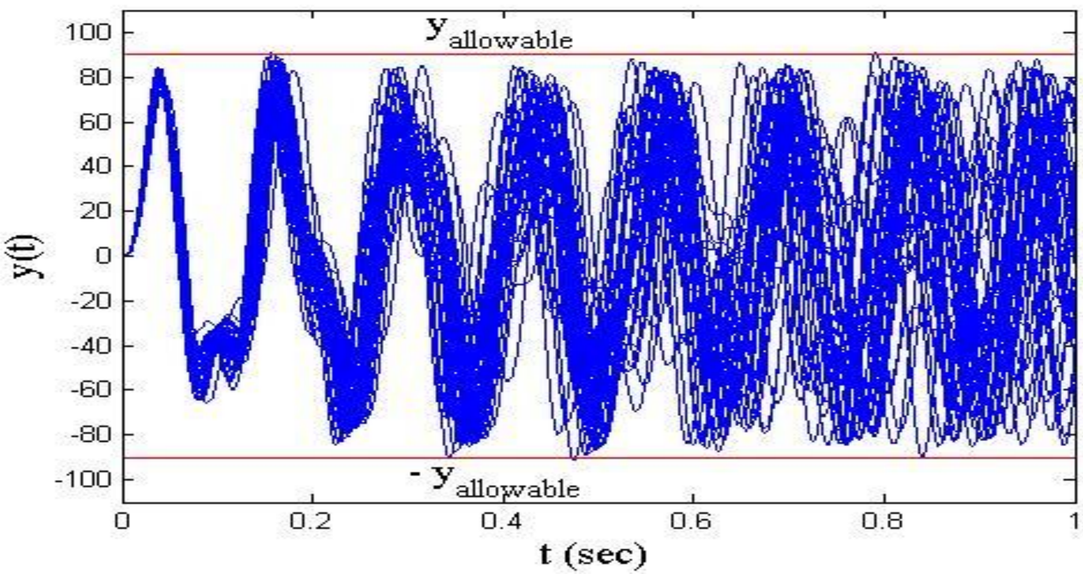
$$m_c \sim N\left(\mu_m, \sigma_m^2\right), \quad \mu_m = 55 \quad Kg, \quad \sigma_m = 5 \quad Kg$$



$$k_s \sim N\left(\mu_k, \sigma_k^2\right), \quad \mu_k = 33E04 \quad N/m$$

$$\sigma_k = 3E04 \quad N/m$$

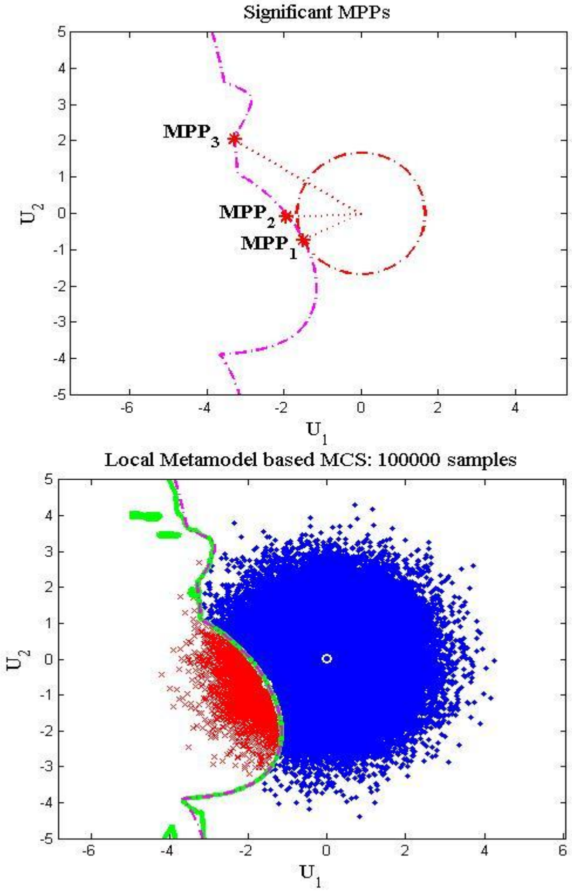
$u(t)$: unit impulse; $\quad 0 \le t \le 5s$

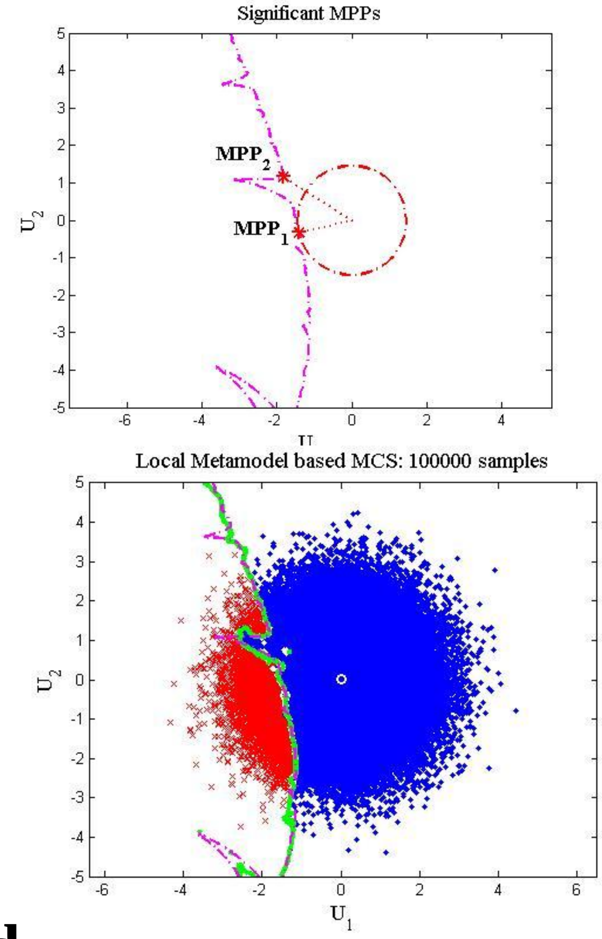


Two-DOF System- Multiple MPPs

T=0.2 sec

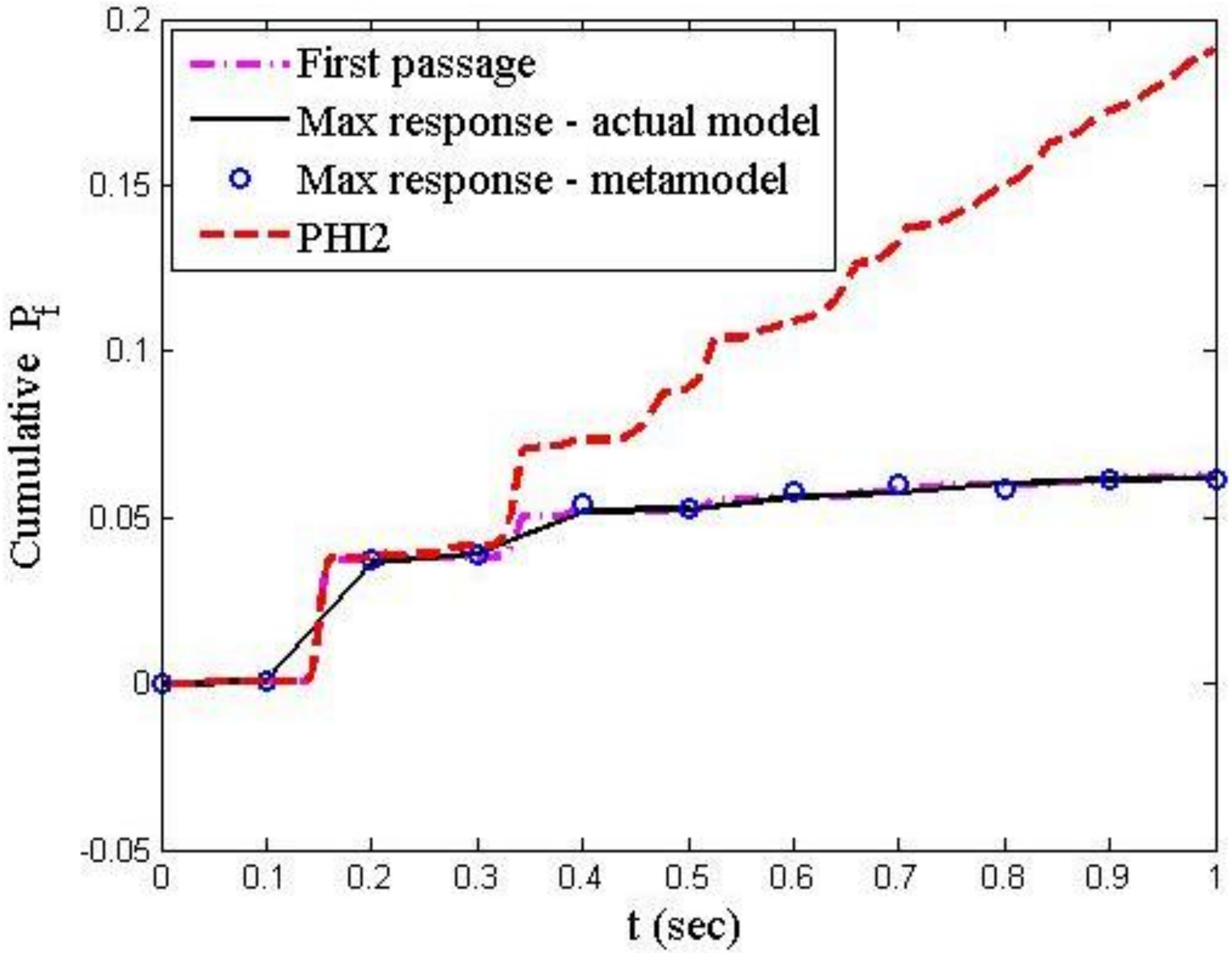
T=1 sec





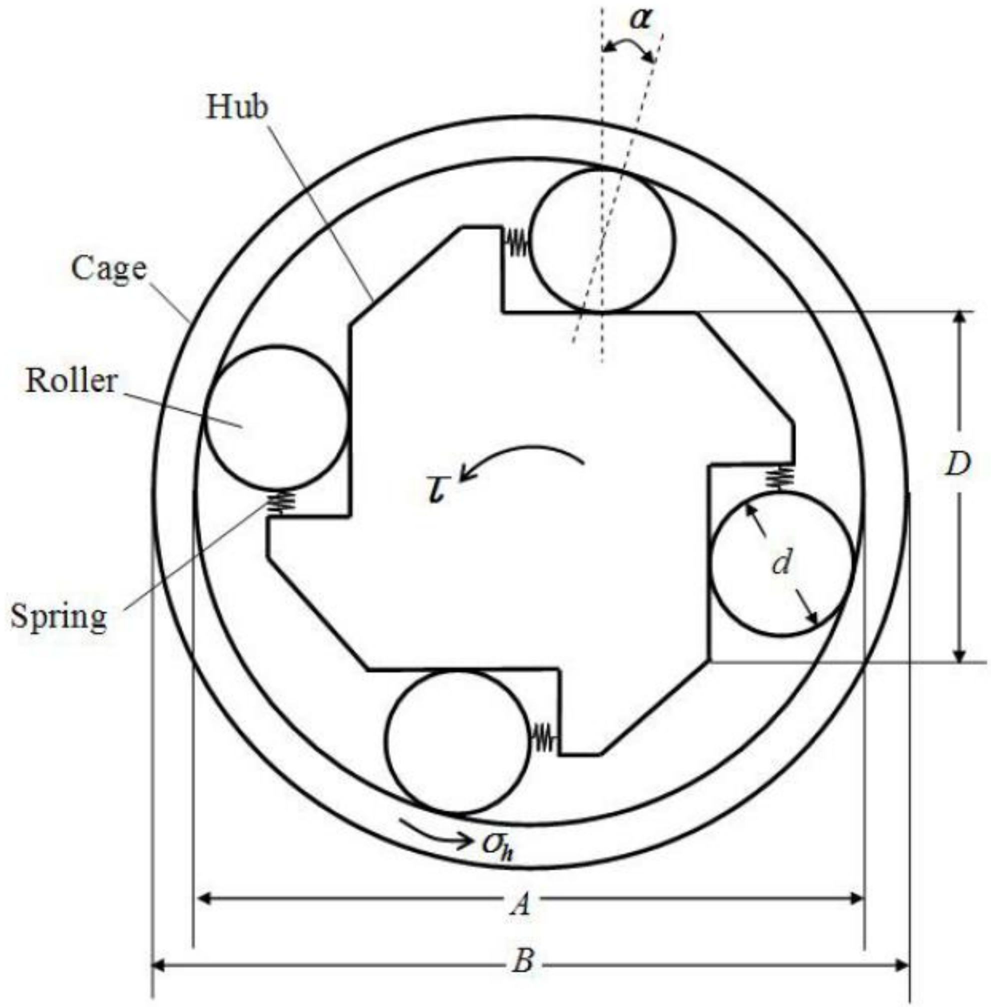
- **Maximum Response method**
- **Niching GA optimization to search for multiple MPPs**

Two-DOF System- Comparison of Pf



Design of a Roller Clutch using Lifecycle Cost

Roller Clutch



Random Design Variables:

D: Hub diameter, mm

d: Roller diameter, mm

A: Cage inner diameter, mm

D, d, and **A** are normally distributed

Due to degradation:

$$\mathbf{D} \rightarrow \mathbf{D}(1-kt)$$

$$\mathbf{d} \rightarrow \mathbf{d}(1-kt)$$

$$\mathbf{A} \rightarrow \mathbf{A}(1+kt)$$

with: $k = 2.5E-04 \quad mm/year$

Roller Clutch

Constraints:

- **Contact angle** $\alpha = 0.11 \pm 0.06 \quad \text{rad}$
- **Torque** $\tau >= 3000 \quad \text{Nm}$
- **Hoop stress** $\sigma_h \le 400 \quad MPa$

$$0.05 \le \cos^{-1}\left(\frac{D-d}{A-d}\right) \le 0.17 \Rightarrow \begin{cases} g_1(D,d,A) = 0.05 - \cos^{-1}\left(\frac{D-d}{A-d}\right) \le 0 \\ g_2(D,d,A) = \cos^{-1}\left(\frac{D-d}{A-d}\right) - 0.17 \le 0 \end{cases}$$

$$g_3(D,d,A) = 3000 - NL\left(\frac{\sigma_c}{c_1}\right)^2 \frac{D^2 d}{4(D+d)}\sqrt{1-S^2} \le 0$$

$$g_4(D,d,A) = \frac{N}{2\pi}\left(\frac{\sigma_c}{c_1}\right)^2\left(\frac{Dd}{(D+d)}\right)\frac{S}{A}\left(\frac{B^2+A^2}{B^2-A^2}\right) - 400E06 \le 0$$

4 Limit States

Roller Clutch: Problem Statement

Minimize
Lifecycle Cost

$$\min_{\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}}} C_L\left(\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}},t_f,r\right)$$

$$\boldsymbol{\sigma}_{\mathbf{X}_L} \le \boldsymbol{\sigma}_{\mathbf{X}} \le \boldsymbol{\sigma}_{\mathbf{X}_U}$$

$$\boldsymbol{\mu}_{\mathbf{X}_L} \le \boldsymbol{\mu}_{\mathbf{X}} \le \boldsymbol{\mu}_{\mathbf{X}_U}$$

s. t.

Case 1

$$F^i\left(\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}},t_0=0\right)= P\left(\bigcup_i^4 \left(g_i(D,d,A,t_0)<0\right)\right) \le p_f\left(t_0=0\right)=0.0013$$

Case 2

$$F^i\left(\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}},t_0=0\right)= P\left(\bigcup_i^4 \left(g_i(D,d,A,t_0)<0\right)\right) \le p_f\left(t_0=0\right)=0.0013$$

$$F^c\left(\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}},t=7.5\right)= P\left(\bigcup_i^4 \left(g_i(D,d,A,t)<0\right)\right) \le p_f\left(t=7.5\right)=0.005$$

Case 3

$$F^c\left(\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\sigma}_{\mathbf{X}},t=10\right)= P\left(\bigcup_i^4 \left(g_i(D,d,A,t)<0\right)\right) \le p_f\left(t=10\right)=0.0716$$

Roller Clutch: Problem Statement

where:

Total Cost, $C_L = C_P + C_I + C_V^E$

$$C_P = \left(3.5 + \frac{0.75}{3\sigma_D}\right) + \left(3.0 + \frac{0.65}{3\sigma_d}\right) + \left(0.5 + \frac{0.88}{3\sigma_A}\right)$$

$$C_I = 20F_Q(\mathbf{X}, t_0)$$

$$C_V^E = \int_0^{t_f} 20e^{-rt} f_R^c(t)\,dt$$

Scrap cost/unit

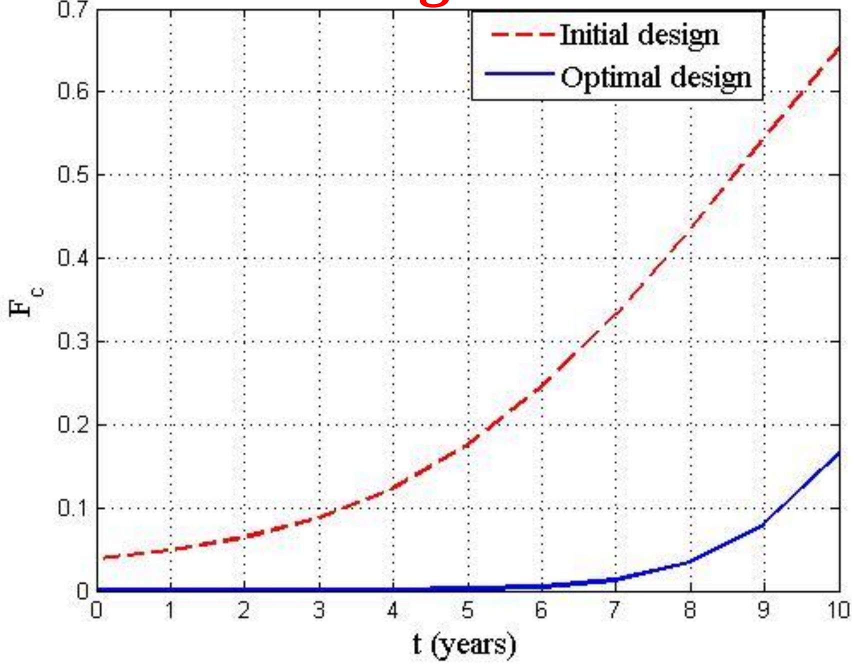
$t_f = 10 \quad years$

$r = 3\%$

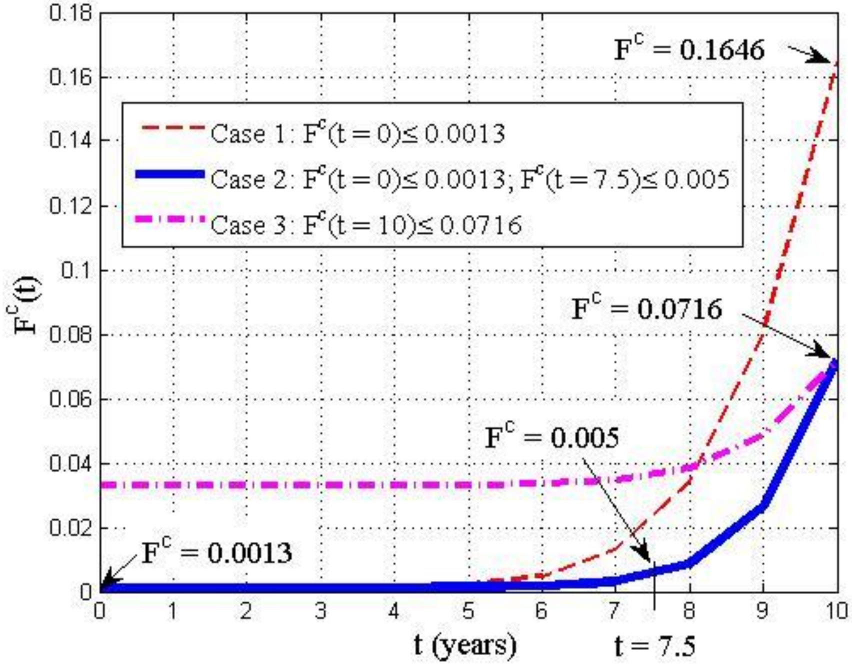
Failure cost/unit (warranty cost)

Roller Clutch: Results

Initial Design vs. Case 1



Case 1 vs. Case 2 and Case 3



		Initial Design	Optimal Design		
			Case 1	Case 2	Case 3
Objective	Total Cost	28.2275	23.876	24.5440	21.1896
	Production Cost	17.3900	21.3340	23.4446	19.9383
	Inspection Cost	0.7677	0.0260	0.0260	0.6596
	Expected Variable Cost	10.0697	2.5161	1.07340	0.5918

Summary/Conclusions

- **A new method to calculate the Cumulative Probability of failure is presented for linear and non-linear problems.**

- **The design study of the roller clutch showed that:**
  - **Lifecycle cost can be reduced by controlling the probability of failure though time.**
  - **Higher lifecycle cost due to higher initial quality does not guarantee acceptable reliability.**

Challenges/Future Work

- Improve further efficiency by:
  - Random process characterization using **time-series modeling** techniques.
  - Solving RBDO problem using **Probabilistic Re-Analysis** which uses a **single MCS**
- Apply presented ideas/approaches to the Army related problems

Q & A

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